

Exercise 33

Find an equation of the tangent line to the curve at the given point.

$$y = \ln(x^2 - 3x + 1), \quad (3, 0)$$

Solution

The aim is to find the slope of the tangent line at $x = 3$, so start by taking the derivative of the function with respect to x by using the chain rule.

$$\begin{aligned} y' &= \frac{d}{dx} \ln(x^2 - 3x + 1) \\ &= \frac{1}{x^2 - 3x + 1} \cdot \frac{d}{dx}(x^2 - 3x + 1) \\ &= \frac{1}{x^2 - 3x + 1} \cdot (2x - 3) \\ &= \frac{2x - 3}{x^2 - 3x + 1} \end{aligned}$$

Set $x = 3$ to get $y'(3)$.

$$y'(3) = \frac{2(3) - 3}{(3)^2 - 3(3) + 1} = \frac{3}{1} = 3$$

Then use the point-slope formula to get the equation of the tangent line.

$$y - 0 = y'(3)(x - 3)$$

$$y - 0 = 3(x - 3)$$

$$y = 3x - 9$$

